Measures and Time

(1) Length

1 centimetre (cm) = 10 millimetres (mm)
1 metre (m) = 100 cm
1 kilometre (km) = 1000 m = 100 000 cm

(2) Capacity

1 litre (*l*) = 1000 millilitres (*ml*)
1 litre (*l*) = 1000 cm^3
1 *ml* = 1 cm^3

(3) Mass

1 gram (g) = 1000 milligrams (mg)
1 kilogram (kg) = 1000 g
1 tonne (t) = 1000 kg

(4) Time

1 hour (h) = 60 minutes (min)
1 min = 60 seconds (s)

Speed

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Indices

Laws of indices

If m, n, a and b are real numbers, then

(a) $a^m \times a^n = a^{m+n}$

(b) $a^m \div a^n = a^{m-n}$

(c) $(a^m)^n = a^{m \times n} = a^{mn}$

(d) $a^m \times b^m = (a \times b)^m$

(e) $a^m \div b^m = \left(\frac{a}{b}\right)^m, b \neq 0$

(f) $a^0 = 1$

(g) $a^{-n} = \frac{1}{a^n}, \frac{1}{a^{-n}} = a^n, a \neq 0$

(h) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

(i) $a^{\frac{1}{n}} = \sqrt[n]{a}, a > 0, n > 0$

(j) $a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}, a > 0, n > 0$

Standard Form

Powers of 10	Name	SI Prefix and Symbol
$1000 = 10^3$	thousand	kilo (k)
$1\ 000\ 000 = 10^6$	million	mega (M)
$1\ 000\ 000\ 000 = 10^9$	billion	giga (G)
$1\ 000\ 000\ 000\ 000 = 10^{12}$	trillion	tera (T)
$0.001 = 10^{-3}$	thousandth	milli (m)
$0.000\ 001 = 10^{-6}$	millionth	micro (μ)
$0.000\ 000\ 001 = 10^{-9}$	billionth	nano (n)
$0.000\ 000\ 000\ 001 = 10^{-12}$	trillionth	pico (p)

Simple and Compound Interest

(1) Simple Interest

$$I = \frac{PRT}{100}$$

Simple interest is the interest calculated on the original principal.

where I = Simple interest,
P = Principal,
R = Rate (per annum) and
T = Time (in years).

Amount = Principal + Interest

(2) Compound Interest

$$A = P(1 + \frac{R}{100})^n$$

In compound interest, the principal changes every year, as the previous year's interest is added onto it.

where A = Amount,
P = Principal,
R = Rate of interest and
n = Number of interest periods.

Compound interest = Amount – Principal

Algebra

Expansion and Factorisation of Algebraic Expressions

1. To expand algebraic expressions, use
   (a) $a(b + c) = ab + ac$
   (b) $a(b - c) = ab - ac$
   (c) $(a + b)^2 = a^2 + 2ab + b^2$
   (d) $(a - b)^2 = a^2 - 2ab + b^2$
   (e) $(a + b)(a - b) = a^2 - b^2$
   (f) $(a + b)(c + d) = ac + ad + bc + bd$

2. To factorise algebraic expressions by:
   (a) extracting common factors,
   $ab + ac = a(b + c)$

   (b) grouping,
   $$\begin{aligned} ax + ay + bx + by &= a(x + y) + b(x + y) \\ &= (x + y)(a + b) \end{aligned}$$

   (c) using algebraic identities,
   $$\begin{aligned} a^2 + 2ab + b^2 &= (a + b)^2 \\ a^2 - 2ab + b^2 &= (a - b)^2 \\ a^2 - b^2 &= (a + b)(a - b) \end{aligned}$$

   (d) inspection.
   E.g. Factorise $2x^2 - x - 6$.

$\times$	x	-2
$2x$	$2x^2$	$-4x$
3	$3x$	-6

   $\therefore 2x^2 - x - 6$
   $= (2x + 3)(x - 2)$

Quadratic Formula

The solutions to the quadratic equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Inequalities

Properties of inequalities

If $a > b$, then

(a) $a + c > b + c$,

(b) $a - c > b - c$,

(c) $ac > bc$ if $c > 0$,

(d) $ac < bc$ if $c < 0$,

(e) $\frac{a}{c} > \frac{b}{c}$ if $c > 0$,

(f) $\frac{a}{c} < \frac{b}{c}$ if $c < 0$,

where a, b and c are real numbers.

Set Language and Notation

$\in$	is an element of
$\notin$	is not an element of
$\subseteq$	is a subset of
$\nsubseteq$	is not a subset of
$\subset$	is a proper subset of
$\not\subset$	is not a proper subset of
$n(A)$	number of elements in set A
ε	Universal set
$\varnothing$ or $\{\ \}$	empty set or null set
A'	complement of set A
$A \cup B$	union of A and B
$A \cap B$	intersection of A and B

Numbers

Integers: ..., −3, −2, −1, 0, 1, 2, 3, ...

Negative integers: −1, −2, −3, −4, −5, ...

Positive integers: 1, 2, 3, 4, 5, ...

Whole numbers: 0, 1, 2, 3, 4, 5, ...

Natural numbers: 1, 2, 3, 4, 5, ...

Perfect squares: 1, 4, 9, 16, 25, 36, 49, ... i.e. $1^2, 2^2, 3^2, 4^2, 5^2, 6^2, 7^2, \ldots$

Cube numbers: 1, 8, 27, 64, 125, ... i.e. $1^3, 2^3, 3^3, 4^3, 5^3 \ldots$

A prime number is a natural number that has exactly 2 different factors, 1 and itself.
Prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, ...

A composite number is a natural number that has more than 2 different factors.
Composite numbers: 4, 6, 8, 9, 10, 12, ...

A **rational number** can be written in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$.

E.g. $-1\frac{1}{2}, -\frac{3}{5}, 8, 9\frac{1}{3}$ are rational numbers.

Matrices

Addition and Subtraction of Matrices

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a-e & b-f \\ c-g & d-h \end{pmatrix}$$

Multiplication of Matrices

$$k\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$$

Congruence and Similarity

(1) Congruent Triangles

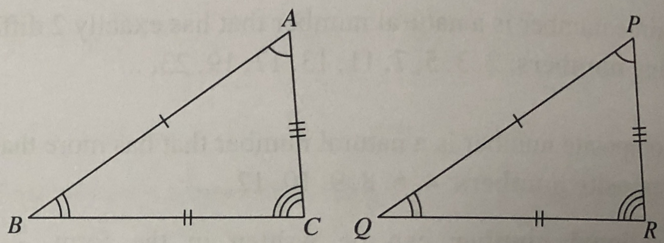
1. Two triangles are **congruent** if they have the **same size** and the **same shape**.

2. If two triangles are congruent, then

   (a) their corresponding angles are equal and
   (b) their corresponding sides are equal.

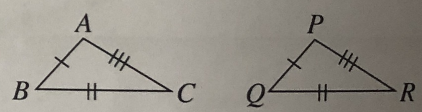
3. If $\triangle ABC \equiv \triangle PQR$, then

$\angle A = \angle P$	$AB = PQ$
$\angle B = \angle Q$	$BC = QR$
$\angle C = \angle R$	$AC = PR$

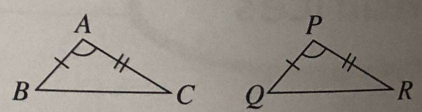


4. One of the following conditions is sufficient for two triangles to be congruent.

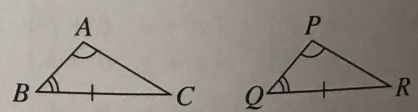
   (a) all three corresponding sides are equal.
   (**SSS** Rule)



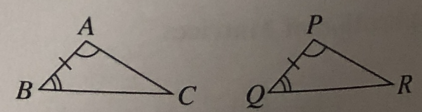
   (b) two corresponding sides and the included angle are equal. (**SAS** Rule)



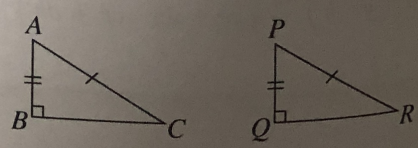
   (c) two angles and a corresponding side are equal.
   (**AAS** Rule)



   (d) two angles and the included side are equal.
   (**ASA** Rule)



   (e) both triangles have a right angle, equal hypotenuse and another equal side.
   (**RHS** Rule)



(2) Similar Triangles

1. Two triangles are **similar** if they have the **same shape** but not necessarily the same size.

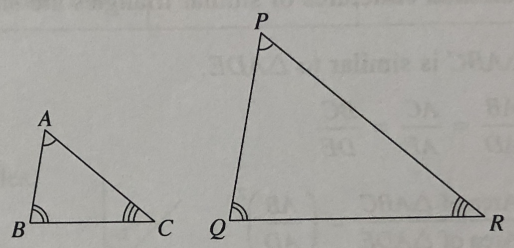
Note! Whenever two figures are congruent, they are also similar. However the converse is not true.

2. If two triangles are similar, then
   (a) their corresponding angles are equal and
   (b) their corresponding sides are in the same ratio.

3. If $\triangle ABC$ is similar to $\triangle PQR$, then

$\angle A = \angle P$
$\angle B = \angle Q$
$\angle C = \angle R$

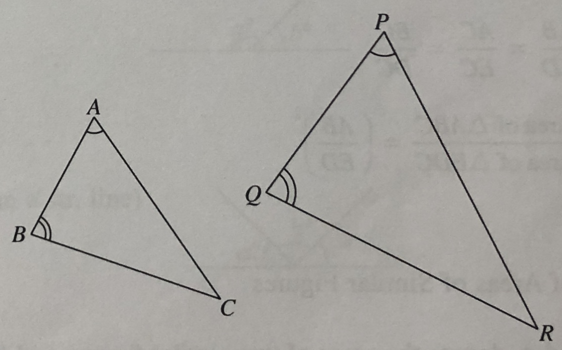
$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$



4. One of the following conditions is sufficient for two triangles to be similar.

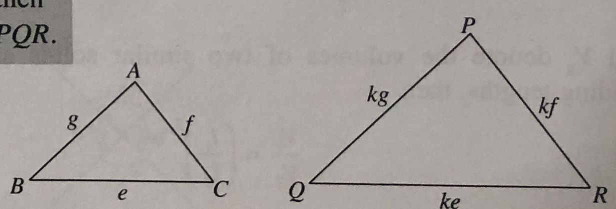
   (a) Two triangles are similar if two of their angles are equal.

If $\angle A = \angle P$ and
$\angle B = \angle Q$ then
$\triangle ABC$ is similar to $\triangle PQR$.



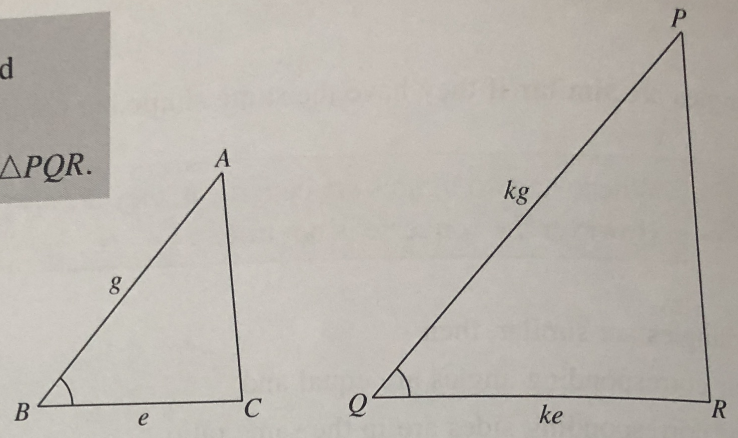
   (b) Two triangles are similar if all three corresponding sides are proportional/in the same ratio.

If $\frac{PQ}{AB} = \frac{QR}{BC} = \frac{PR}{AC} = k$,
where k is a constant, then
$\triangle ABC$ is similar to $\triangle PQR$.



(c) Two triangles are similar if two of their sides are proportional and the included angle is equal.

If $\frac{PQ}{AB} = \frac{QR}{BC} = k$ and
$\angle B = \angle Q$, then
$\triangle ABC$ is similar to $\triangle PQR$.

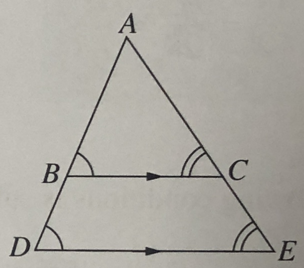


5. Two common structures of similar triangles are shown below.

(a) $\triangle ABC$ is similar to $\triangle ADE$.

$$\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$$

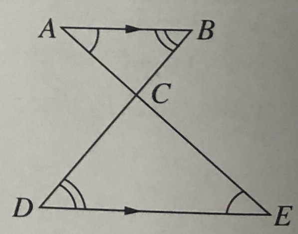
$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle ADE} = \left(\frac{AB}{AD}\right)^2$$



(b) $\triangle ABC$ is similar to $\triangle EDC$.

$$\frac{AB}{ED} = \frac{AC}{EC} = \frac{BC}{DC}$$

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle EDC} = \left(\frac{AB}{ED}\right)^2$$



6. Ratio of Areas of Similar Figures

If A_1 and A_2 denote the areas of two similar figures and l_1 and l_2 denote their corresponding lengths, then

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

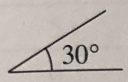
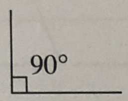
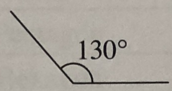
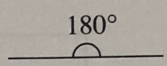
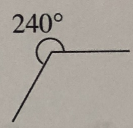
7. Ratio of Volumes of Similar Solids

If V_1 and V_2 denote the volumes of two similar solids and l_1 and l_2 denote their corresponding lengths, then

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

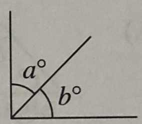
Angles, Triangles, Quadrilaterals

(1) Types of Angles

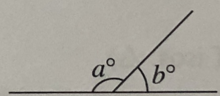
Acute angle	Right angle	Obtuse angle	Straight angle	Reflex angle
				
Angles less than 90°.	Angles equal to 90°.	Angles greater than 90° but less than 180°.	Angles equal to 180°.	Angles greater than 180° but less than 360°.

(2) Geometrical Properties of Angles

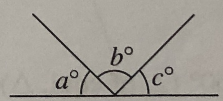
1. $a° + b° = 90°$ (Complementary angles)



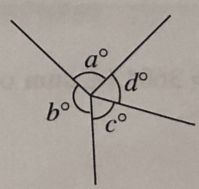
2. $a° + b° = 180°$ (Supplementary angles)



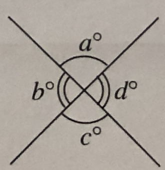
3. $a° + b° + c° = 180°$ (adj. ∠s on a str. line)



4. $a° + b° + c° + d° = 360°$ (∠s at a point)



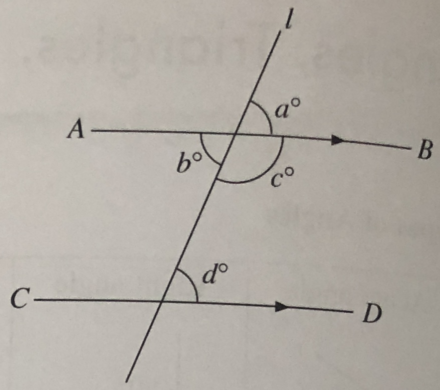
5. $a° = c°$ (vert. opp. ∠s)
   $b° = d°$ (vert. opp. ∠s)



6. $a° = d°$ (corr. ∠s, $AB // CD$)

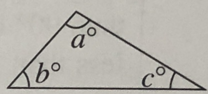
$b° = d°$ (alt. ∠s, $AB // CD$)

$c° + d° = 180°$ (int. ∠s, $AB // CD$)

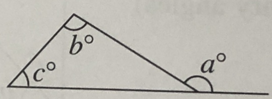


(3) Angle Properties of Triangles and Quadrilaterals

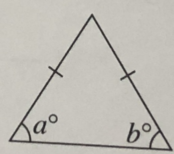
1. $a° + b° + c° = 180°$ (∠ sum of △)



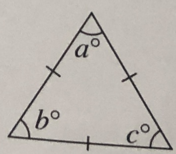
2. $a° = b° + c°$ (ext. ∠ of △)



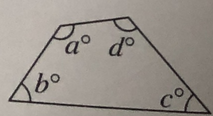
3. $a° = b°$ (base ∠s of isos. △)



4. $a° = b° = c° = 60°$ (∠ of equi. △)



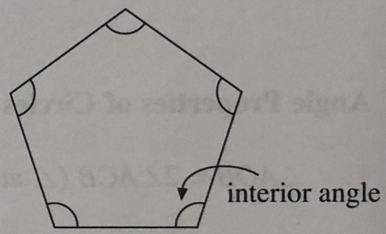
5. $a° + b° + c° + d° = 360°$ (∠ sum of quad.)



Polygons

Number of sides	Name of polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

For an n-sided polygon, sum of interior angles
$= (n - 2) \times 180°$

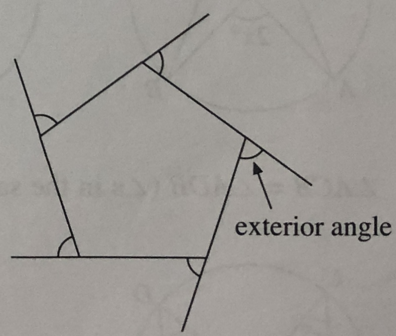


For a **regular** n-sided polygon, size of each interior angle
$= \dfrac{(n - 2) \times 180°}{n}$

Sum of all the exterior angles of any polygon = 360°

For a **regular** n-sided polygon,

size of each exterior angle $= \dfrac{360°}{n}$

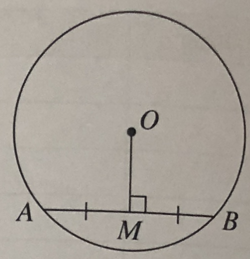


Properties of Circles

(1) Symmetrical Properties of Circles

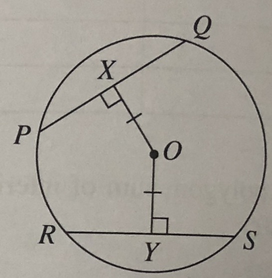
1. If $OM \perp AB$, then $AM = MB$.

   Conversely, if $AM = MB$, then $OM \perp AB$.



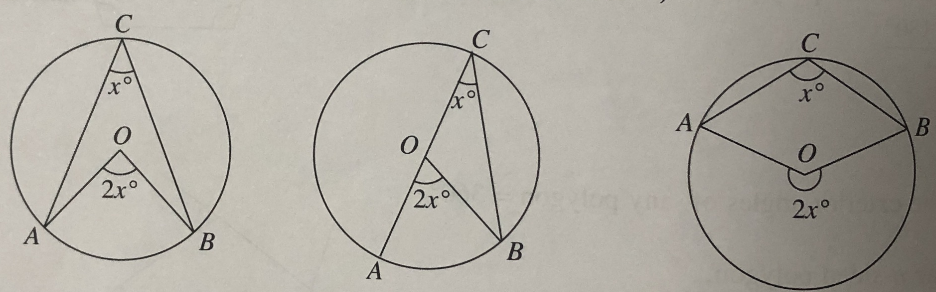
2. If $PQ = RS$, then $OX = OY$.

   Conversely, if $OX = OY$, then $PQ = RS$.

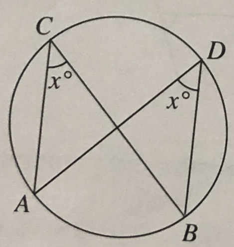


(2) Angle Properties of Circles

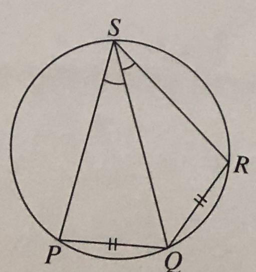
1. $\angle AOB = 2\angle ACB$ ($\angle$ at centre = 2 $\angle$ at circumference)



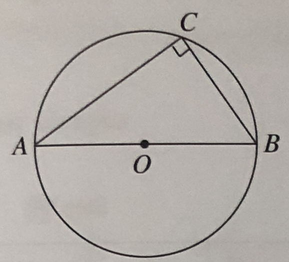
2. $\angle ACB = \angle ADB$ ($\angle$s in the same segment)



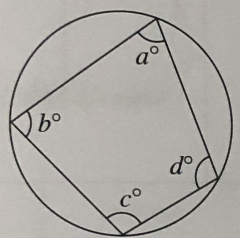
Note: If $PQ = QR$, then $\angle PSQ = \angle QSR$ ($\angle$s in the same segment)



3. $\angle ACB = 90°$ (rt. $\angle$ in semicircle)

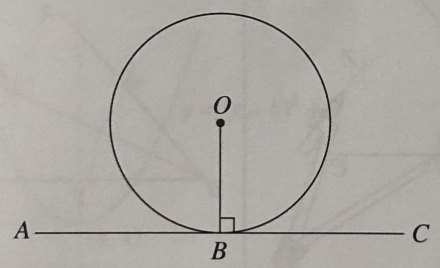


4. $a° + c° = 180°$ ($\angle$s in opp. segments)
$b° + d° = 180°$ ($\angle$s in opp. segments)

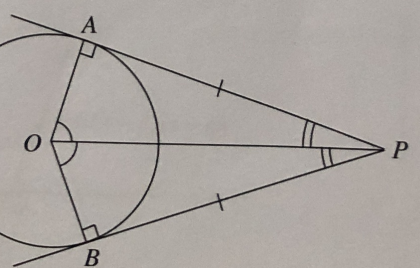


(3) Tangent Theorems

1. If ABC is a tangent to the circle at B, then
$\angle OBA = \angle OBC = 90°$ (tan $\perp$ rad.)

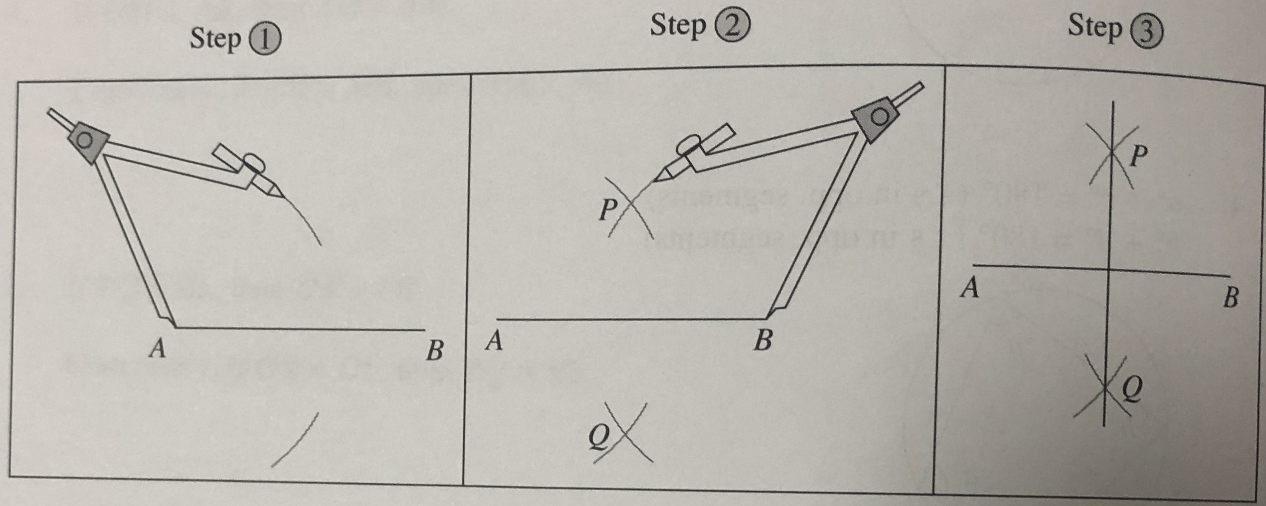


2. If PA and PB are tangents to the circle, then
(a) $PA = PB$ (tangents from an external point),
(b) $\angle POA = \angle POB$,
(c) $\angle APO = \angle BPO$.

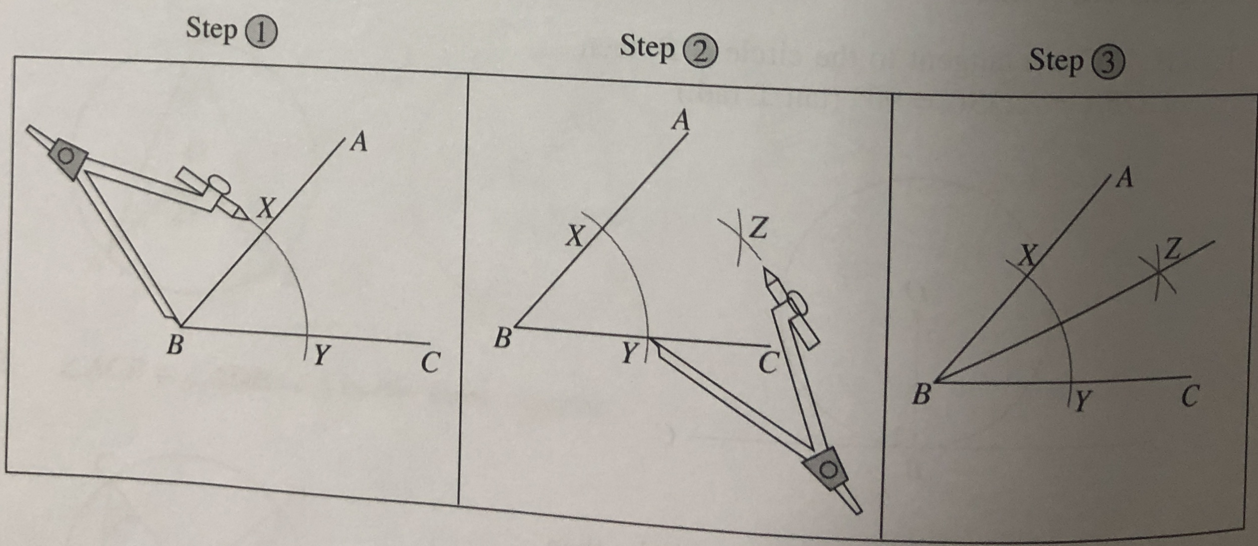


Simple Constructions

(1) To construct the perpendicular bisector of the line *AB*:

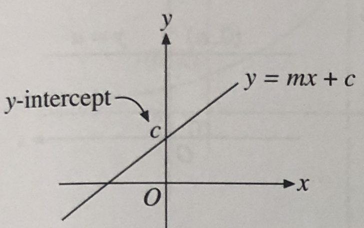


(2) To construct the angle bisector of $\angle ABC$:

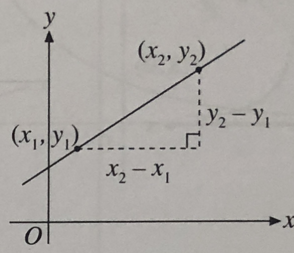


Graphs and Graphical Solutions of Equations

(1) Linear Graphs

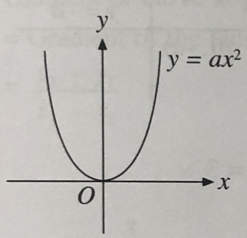


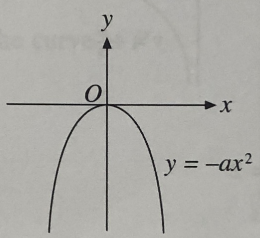
m = gradient
c = y-intercept

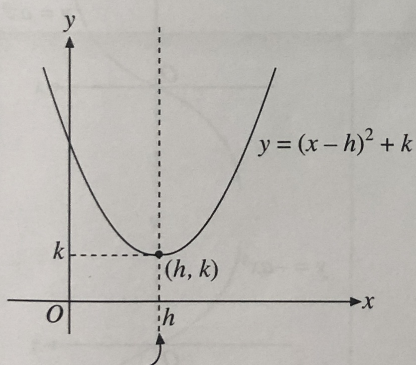


$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

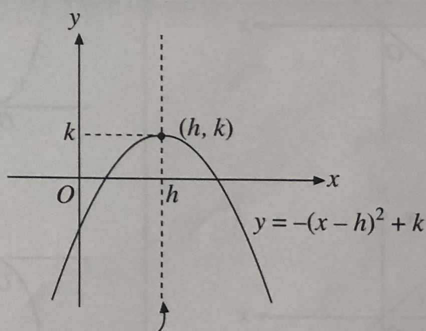
(2) Quadratic Graphs



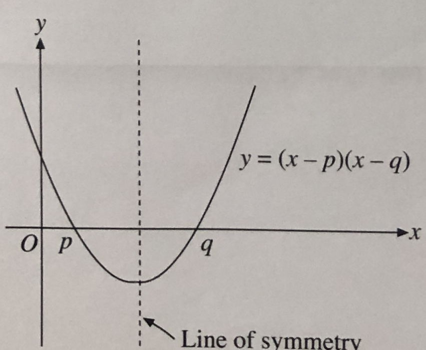




Line of symmetry
$x = h$
Minimum point (h, k)

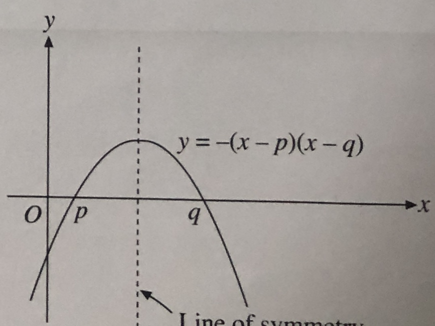


Line of symmetry
$x = h$
Maximum point (h, k)



Line of symmetry

$$x = \frac{p+q}{2}$$

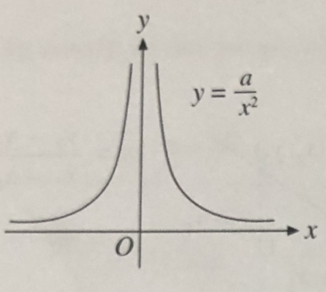


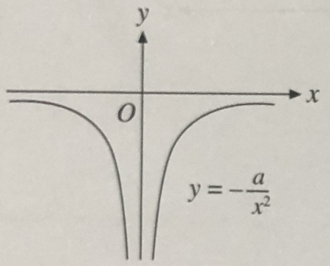
Line of symmetry

$$x = \frac{p+q}{2}$$

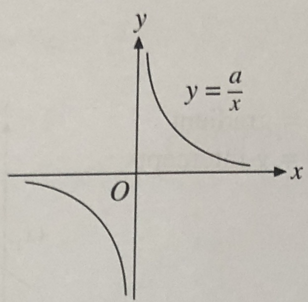
(3) Graphs of Power Functions $y = ax^n$

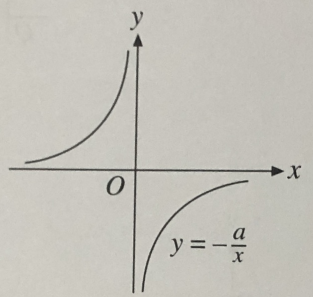
$n = -2$



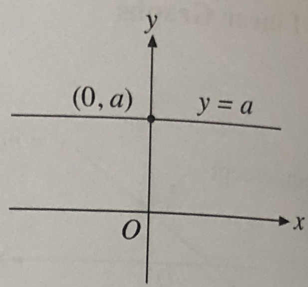


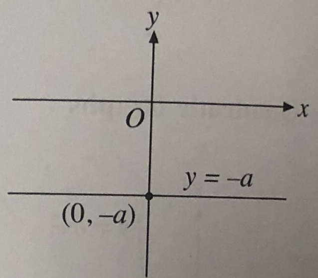
$n = -1$



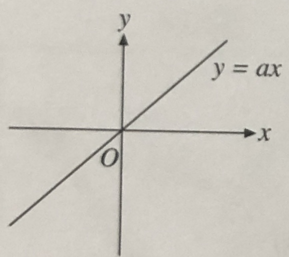


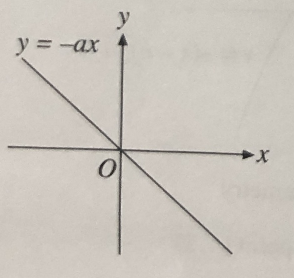
$n = 0$



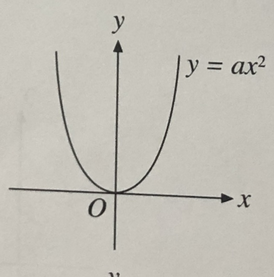


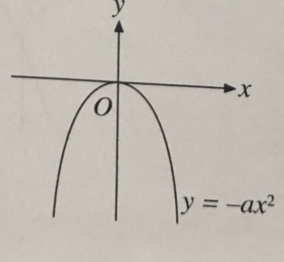
$n = 1$



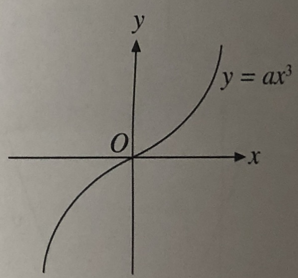


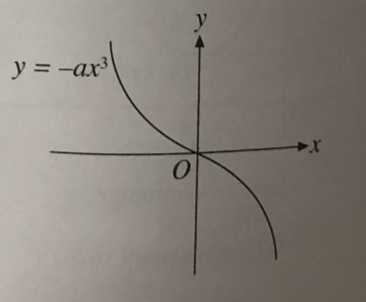
$n = 2$



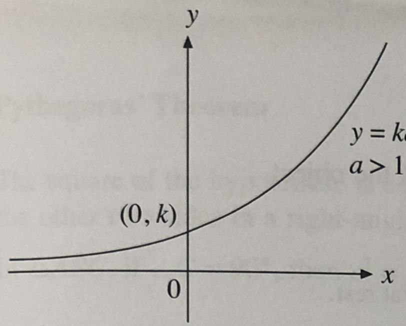


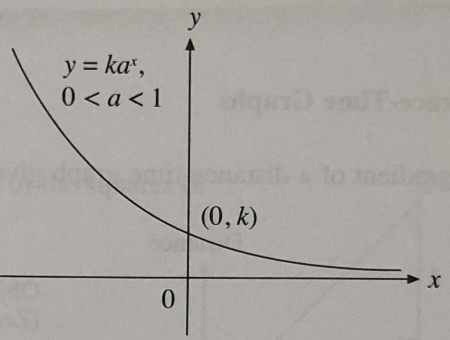
$n = 3$





(4) Graphs of Exponential Functions $y = ka^x$





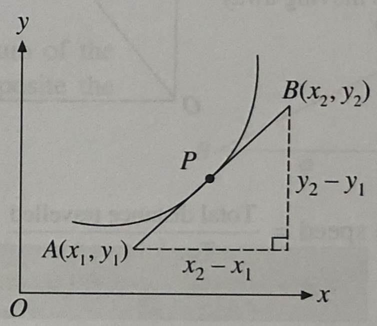
Here $a > 0$ and $a \neq 1$ and k is a constant.

(5) Gradient of Curves

Gradient of curve at point P

= Gradient of the tangent to the curve at P

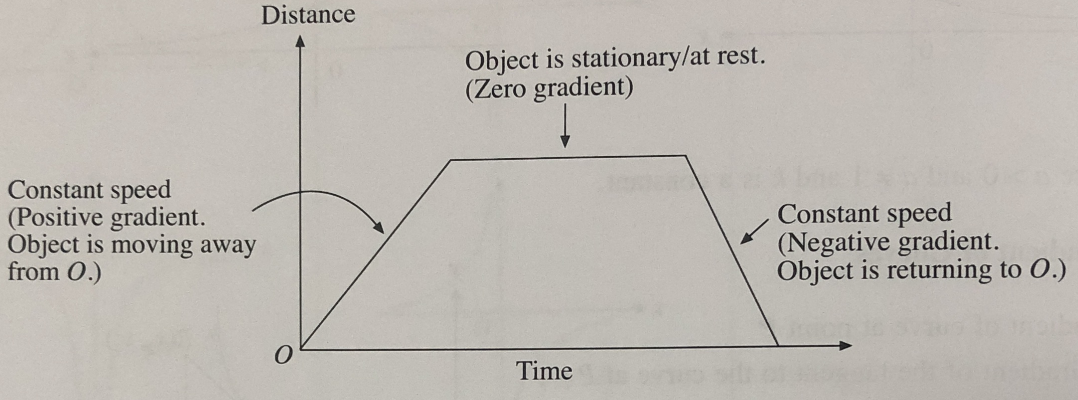
$= \dfrac{y_2 - y_1}{x_2 - x_1}$



Distance-Time and Speed-Time Graphs

(1) Distance-Time Graphs

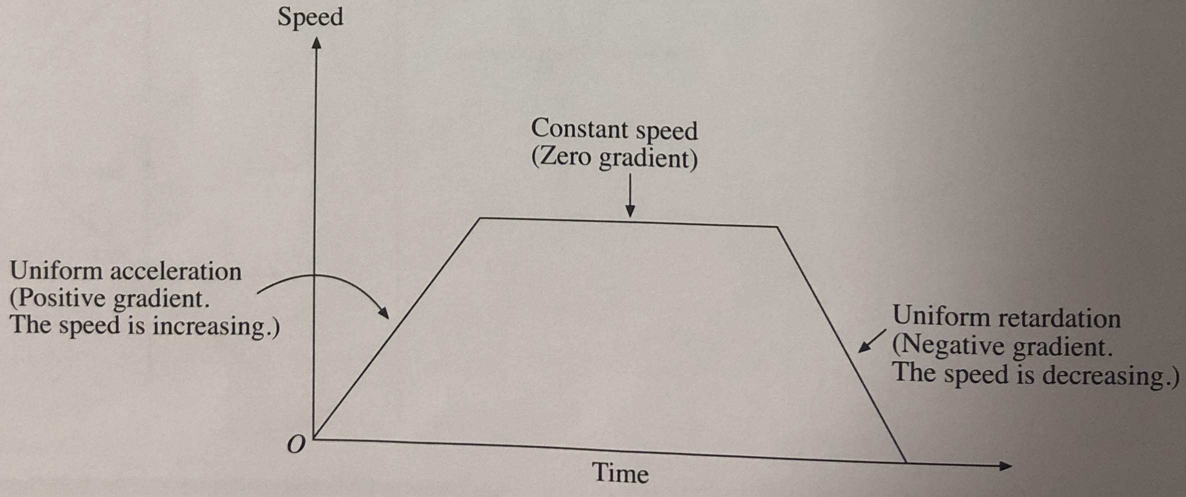
The gradient of a distance-time graph gives the **speed** of the object.



$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

(2) Speed-Time Graphs

The gradient of a speed-time graph gives the **acceleration** of the object.



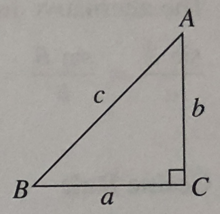
Distance travelled = Area under the graph

Pythagoras' Theorem

(1) Pythagoras' Theorem

The square of the hypotenuse is equal to the sum of the squares of the other two sides in a right-angled triangle.

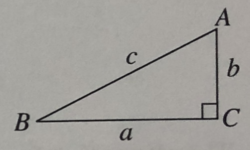
In $\triangle ABC$, if $\angle C = 90°$, then $c^2 = a^2 + b^2$.



(2) Converse of the Pythagoras' Theorem

If the square of the longest side is equal to the sum of the squares of the other two sides, then the angle opposite the longest side is a right angle.

In $\triangle ABC$, if $c^2 = a^2 + b^2$, then $\angle C = 90°$.



Note! The hypotenuse is the longest side which is opposite the right angle.

Trigonometry

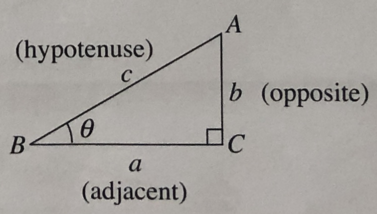
(1) Trigonometric Ratios for Acute Angles

For a right-angled triangle *ABC*,

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{b}{a}$$



(2) Trigonometric Identities for Obtuse Angles

$\sin(180° - \theta) = \sin\theta$

$\cos(180° - \theta) = -\cos\theta$

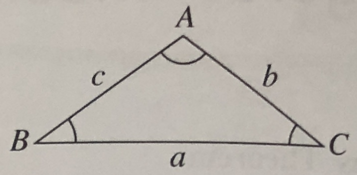
In general, if θ is acute or obtuse:

$\sin\theta = \sin(180° - \theta)$

$\cos\theta = -\cos(180° - \theta)$

(3) The Sine Rule

1. In any triangle ABC, the Sine Rule states that:



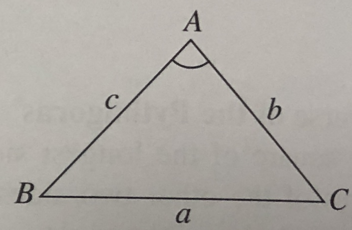
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

2. The alternative form of the Sine Rule to find an angle is given below.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

(4) The Cosine Rule

1. In any triangle ABC, the Cosine Rule states that:



$$a^2 = b^2 + c^2 - 2bc \cos A$$

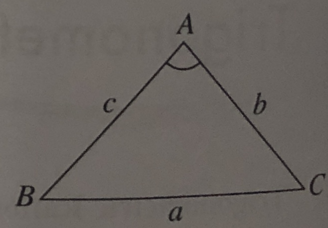
2. The alternative form of the Cosine Rule is given below.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

(5) Area of a Triangle

To find the area of a triangle given 2 sides and the included angle, use:

Area of $\triangle ABC = \frac{1}{2} bc \cos A$

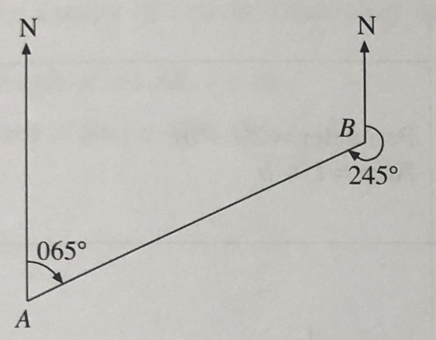


Bearing

1. A **bearing** is an angle that tells the direction of one place from another.

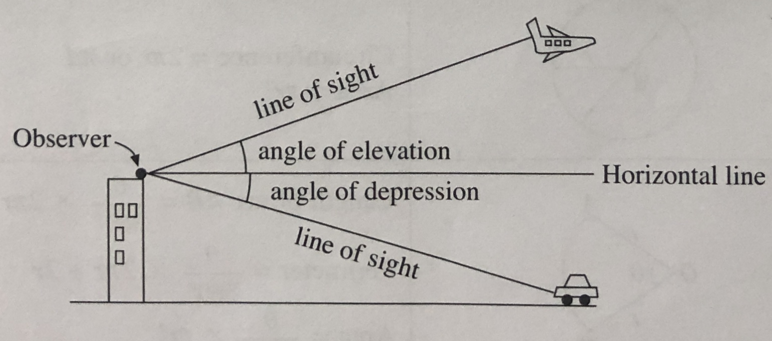
2. A bearing is
   (a) measured from the **north**,
   (b) measured in a **clockwise** direction and
   (c) written as a **three-digit** number. (000° to 360°)

E.g.



The bearing of B from A is 065°.
The bearing of A from B is 245°.

Angle of Elevation and Depression

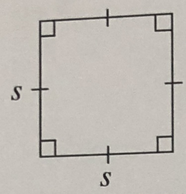
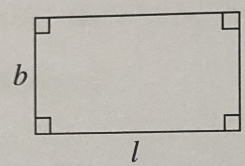
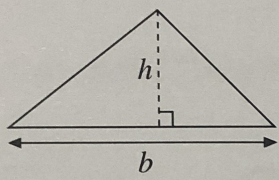
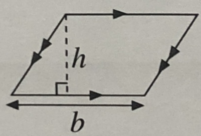
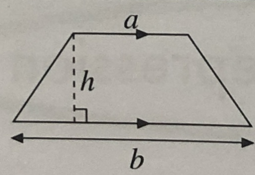
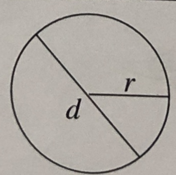
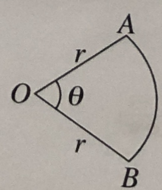


1. When you look at the airplane, the angle between the line of sight and the horizontal line is called the **angle of elevation**.

2. When you look down at the car, the angle between the horizontal line and the line of sight is called the **angle of depression**.

Mensuration

(1) Perimeters and Areas of Plane Figures

Name	Figure	Perimeter or Area
Square		Perimeter = $4s$ Area = s^2
Rectangle		Perimeter = $2(l + b)$ Area = $l \times b$
Triangle		Area = $\frac{1}{2} \times b \times h$
Parallelogram		Area = $b \times h$
Trapezium		Area = $\frac{1}{2} \times (a + b) \times h$
Circle		Circumference = $2\pi r$ or πd Area = πr^2
Sector		Length of arc $AB = \frac{\theta}{360°} \times 2\pi r$ Perimeter = $\frac{\theta}{360°} \times 2\pi r + 2r$ Area = $\frac{\theta}{360°} \times \pi r^2$

(2) Arc length, Sector Area and Area of a Segment in Radian Measure

1. The formula to convert radians to degrees and vice-versa are given below.

π rad = 180°

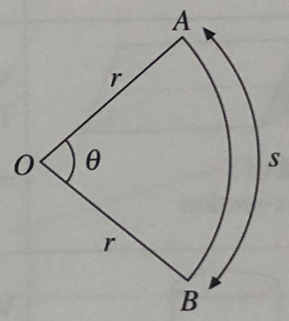
$1 \text{ rad} = \dfrac{180°}{\pi}$

$1° = \dfrac{\pi}{180}$ rad

2. For a sector of a circle, subtending an angle θ radians at centre O, radius r:

Length of arc AB, $s = r\theta$

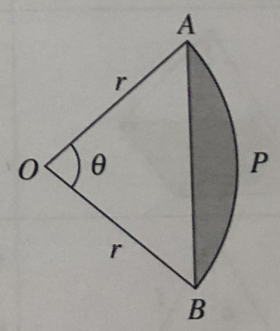
Area of sector AOB, $A = \dfrac{1}{2}r^2\theta$



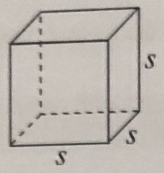
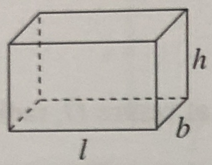
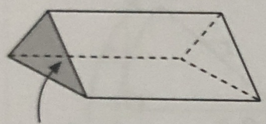
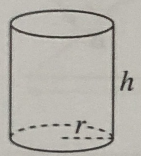
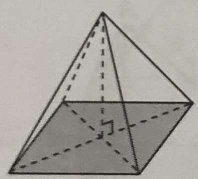
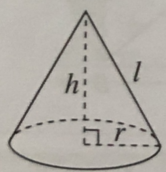
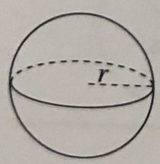
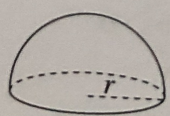
Area of segment APB

= Area of sector AOB – Area of $\triangle AOB$

$= \dfrac{1}{2}r^2\theta - \dfrac{1}{2}r^2 \sin \theta$



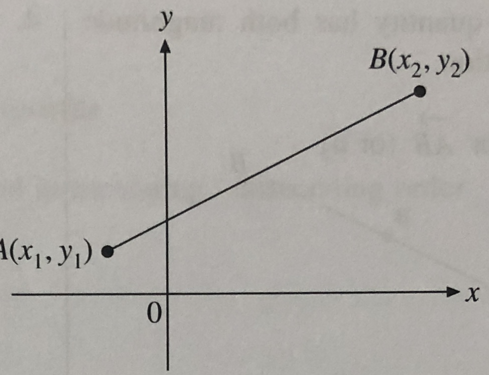
(3) Volumes and Surface Areas of Solids

Name	Solid	Volume / Surface Area
Cube		Volume = s^3 Total surface area = $6s^2$
Cuboid		Volume = $l \times b \times h$ Total surface area = $2(lb + lh + bh)$
Prism	 Area of cross-section	Volume = Area of cross-section $\times$ Length Total surface area $= \left(\text{Perimeter of cross-section}\right) \times \text{Length} + 2 \times \left(\text{Area of cross-section}\right)$
Cylinder		Volume = $\pi r^2 h$ Curved surface area = $2\pi rh$ Total surface area = $2\pi r^2 + 2\pi rh$
Pyramid		Volume = $\frac{1}{3} \times$ Base area $\times$ Height
Cone		Volume = $\frac{1}{3}\pi r^2 h$ Curved surface area = πrl Total surface area = $\pi r^2 + \pi rl$
Sphere		Volume = $\frac{4}{3}\pi r^3$ Surface area = $4\pi r^2$
Hemisphere		Volume = $\frac{2}{3}\pi r^3$ Curved surface area = $2\pi r^2$ Total surface area = $\pi r^2 + 2\pi r^2$ $= 3\pi r^2$

Coordinate Geometry

Length of $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

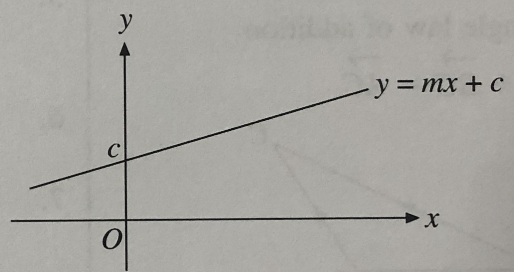
Gradient, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$



Equation of a straight line:

$y = mx + c$

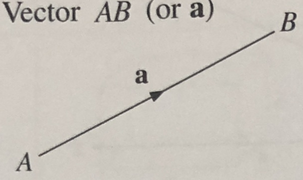
where m = gradient and c = y-intercept.



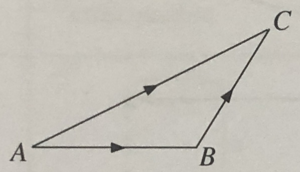
Vectors

1. A vector quantity has both magnitude and direction.

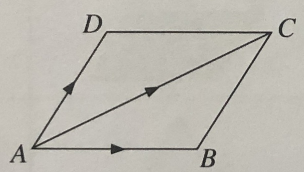
   E.g. Vector $\overrightarrow{AB}$ (or **a**)



2. To add 2 vectors, use:
   (a) Triangle law of addition
   $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

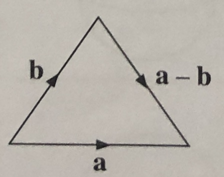


   (b) Parallelogram law of addition
   $\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$

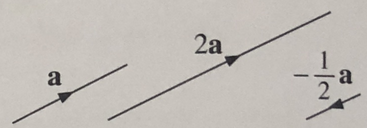


3. To subtract 2 vectors, add the first vector to the negative of the second vector.

   $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$



4. When a vector **a** is multiplied by a scalar k,
   (a) **a** and k**a** are in the **same** direction and $|k\mathbf{a}| = k|\mathbf{a}|$ if $k > 0$.
   (b) **a** and k**a** are in **opposite** directions and $|k\mathbf{a}| = k|\mathbf{a}|$ if $k < 0$.



5. Vector **a** is parallel to vector **b** if and only if $\mathbf{a} = k\mathbf{b}$.

6. If $\overrightarrow{AB} = \overrightarrow{CD}$, then $AB \parallel CD$ and $AB = CD$.

7. If $\overrightarrow{PQ} = k\overrightarrow{PR}$, then $PQ \parallel PR$ and $PQ = kPR$. P is a common point. The points P, Q and R lie in a straight line (i.e. they are collinear).

8. $\overrightarrow{OA}$ is the position vector of A relative to the origin O.

9. For any two points A and B, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$.

10. If A is the point (p, q), then
    (a) $\overrightarrow{OA} = \begin{pmatrix} p \\ q \end{pmatrix}$,
    (b) $|\overrightarrow{OA}| = \sqrt{p^2 + q^2}$.

11. If $\mathbf{a} = \begin{pmatrix} p \\ q \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} r \\ s \end{pmatrix}$, then
    (a) $\mathbf{a} + \mathbf{b} = \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p + r \\ q + s \end{pmatrix}$,
    (b) $\mathbf{a} - \mathbf{b} = \begin{pmatrix} p \\ q \end{pmatrix} - \begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} p - r \\ q - s \end{pmatrix}$,
    (c) $m\mathbf{a} = m\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} mp \\ mq \end{pmatrix}$.

Statistics

Range = Largest value – Smallest value

Interquartile range = Upper quartile – Lower quartile

Median = Middle value of a set of data arranged in ascending / descending order

Mode = The value that occurs most frequently

Ungrouped Data

Mean = $\frac{x_1 + x_2 + \dots x_n}{n}$

Standard deviation = $\sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$

Grouped Data

Mean = $\frac{\Sigma fx}{\Sigma f}$ where x = mid-value of class interval and f = frequency of class interval.

Standard deviation = $\sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$

(A) Using a Calculator to compute the Mean and Standard Deviation for Ungrouped Data

Example

The length of 6 fishes, in centimetres, are given below.

36, 31, 28, 43, 39, 30

Use a calculator to compute the mean and the standard deviation.

Solution

Note! Always remember to clear all the data currently stored in the calculator memory by pressing [SHIFT] [9] (CLR) [2] [=] [AC].

Steps to find the mean and standard deviation.

1. [MODE]
2. [2] (STAT) ← Changes the calculator to the statistics mode (STAT).
3. [1] (1-VAR) ← Only one variable, i.e. the x variable.
4. Enter the data, one at a time.

   [3] [6] [=]
   [3] [1] [=]
   [2] [8] [=]
   [4] [3] [=]
   [3] [9] [=]
   [3] [0] [=]

	x	FREQ
1	36	1
2	31	1
3	28	1
4	43	1
5	39	1
6	30	1

5. [AC]
6. [SHIFT] [1]
7. [4] (VAR)
8. [2] $\bar{x}$ ← To find the mean, $\bar{x}$.
9. [=] ← The screen displays the value of the mean, i.e. 34.5

10. [AC]

11. [SHIFT] [1]

12. [4] (VAR)

13. [3] (σx) ← To find the standard deviation.

14. [=] ← The screen displays the value of the standard deviation, i.e. 5.315072906

Mean = 34.5 cm
Standard deviation = 5.32 cm (correct to 3 sig. fig.)

Teacher's Tip

To compute the standard deviation only, follow steps 1 – 7, then jump to steps 13 and 14.

(B) Using a Calculator to compute the Mean and Standard Deviation for Grouped Data

Example

The table below shows the waiting times of 50 patients to see a doctor at a clinic.

Time (min)	$10 < x \leqslant 15$	$15 < x \leqslant 20$	$20 < x \leqslant 25$	$25 < x \leqslant 30$	$30 < x \leqslant 35$
Frequency	8	10	15	12	5

Use a calculator to compute the mean and the standard deviation.

Solution

Time (min)	$10 < x \leqslant 15$	$15 < x \leqslant 20$	$20 < x \leqslant 25$	$25 < x \leqslant 30$	$30 < x \leqslant 35$
Frequency	8	10	15	12	5
Mid-value (x)	12.5	17.5	22.5	27.5	32.5

Note! Always remember to clear all the data currently stored in the calculator memory by pressing [SHIFT] [9] (CLR) [2] [=] [AC].

Steps to find the mean and standard deviation.

1. [SHIFT] [MODE] [▼] [4] (STAT) [1] (ON) ← Switch on FREQ column to input the frequency values.
2. [MODE]
3. [2] (STAT) ← Changes the calculator to the statistics mode (STAT).
4. [1] (1-VAR)
5. Enter the data, for each class first, by keying in the mid-value (x) column first. Then use the [▼] [▶] move to the frequency (f) column to key in the frequencies.

x	FREQ
[1] [2] [·] [5] [=]	[8] [=]
[1] [7] [·] [5] [=]	[1] [0] [=]
[2] [2] [·] [5] [=]	[1] [5] [=]
[2] [7] [·] [5] [=]	[1] [2] [=]
[3] [2] [·] [5] [=]	[5] [=]

	x	FREQ
1	12.5	8
2	17.5	10
3	22.5	15
4	27.5	12
5	32.5	5

6. [AC]

7. [SHIFT] [1]

8. [4] (VAR)

9. [2] $\bar{x}$ ← To find the mean, $\bar{x}$.

10. [=] ← The screen displays the value of the mean, 22.1.

11. [AC]

12. [SHIFT] [1]

13. [4] (VAR)

14. [3] (σx) ← To find the standard deviation.

15. [=] ← The screen displays the value of the standard deviation, i.e. 6.069596362.

Mean = 22.1 min
Standard deviation = 6.07 min (correct to 3 sig. fig.)

Teacher's Tip

1. To compute the standard deviation only, follow steps 1 – 8, then jump to steps 14 and 15.
2. We can also get the values of Σfx and Σfx^2 by pressing the following keys:
   [AC] [SHIFT] [1] (STAT) [3] (Sum) [2] (Σx) [=] and
   [AC] [SHIFT] [1] (STAT) [3] (Sum) [1] (Σx^2) [=]
   respectively.